

Nonlinear Frequency Response Analysis

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INTES

INTES Ingenieurgesellschaft für
technische Software mbH



Privately held and independent Finite Element Technology company since 1984 located in Stuttgart, Paris, Tokyo and Ann Arbor.

*Developing and marketing of the FE analysis software **PERMAS** with graphical user interface **VisPER**.*

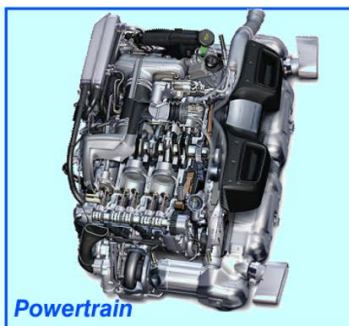
Unified software for thermo-mechanics, vibro-acoustics, with built-in optimization and fatigue analysis.

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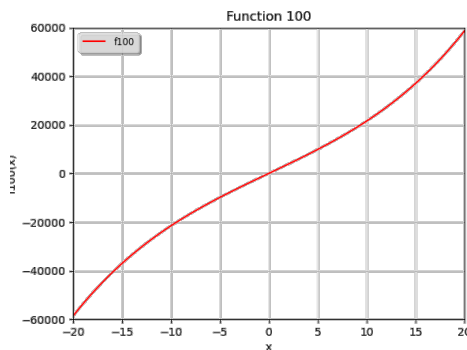


Power tools

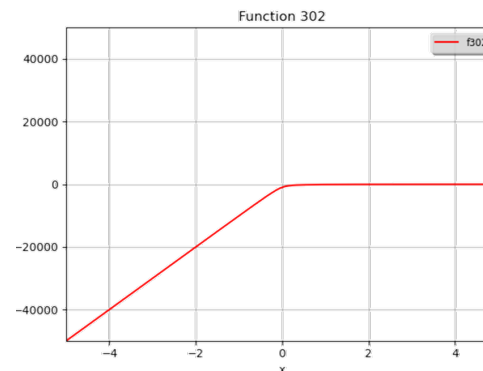


Ships

- Result relevant nonlinearities must be considered in dynamic analyses.
- Time domain analysis is established to consider nonlinearities in dynamic analyses.
- Frequency domain analyses with nonlinearities are often requested as an analogue to the linear frequency response.
- **Harmonic Balance Method (HBM)** as dynamic response analysis in frequency domain for linear models **with local nonlinearities** is available in PERMAS.
- Local nonlinearities are for example **nonlinear mounts from rubber materials** or **contacts**.



Nonlinear
rubber
material



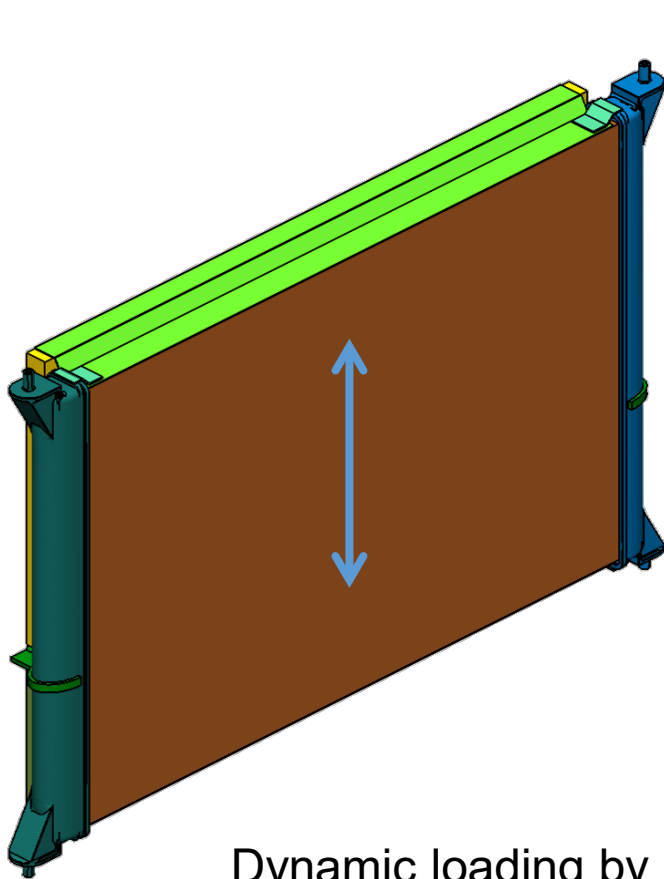
Nonlinear
contact

Accompanying example radiator modul under dynamic loading:

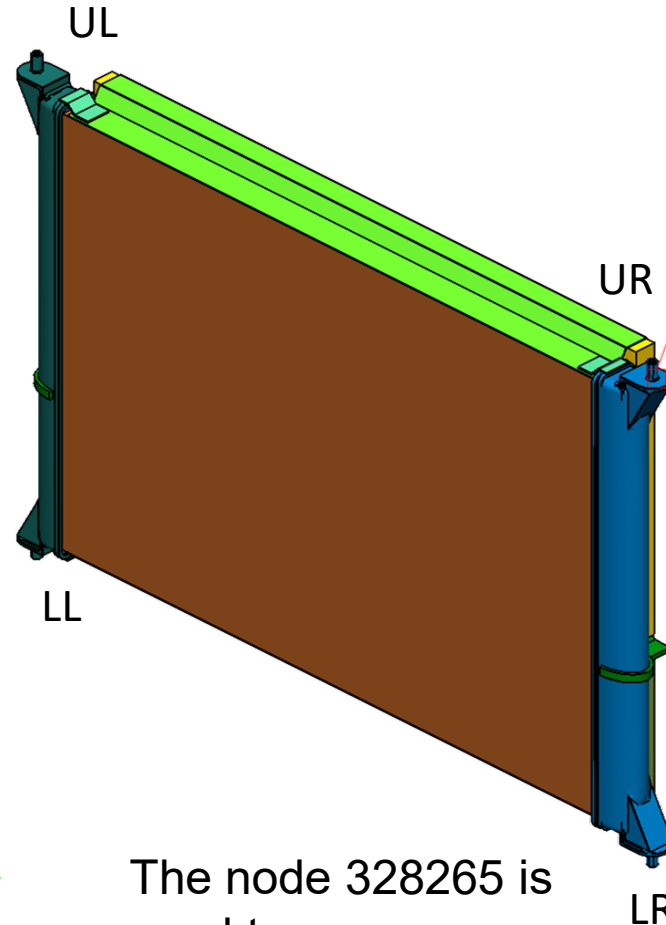
1. Model, loading, boundary conditions
 - Rubber mounting material by simulation
2. Radiator with rubber mounting
 - Eigenfrequencies
 - Linear frequency response
 - HBM
 - Increase damping
3. Add contact – comparison vs. without contact
 - Transformation to time domain
 - Check of forces and displacement at mounting

The Radiator Model

- Nodes: 405,000
- Elements: 750,000
 - (mainly TET4, HEXE8)
- DOFs: 1,174,000
- Materials: 3
 - (plastic, alu, steel)

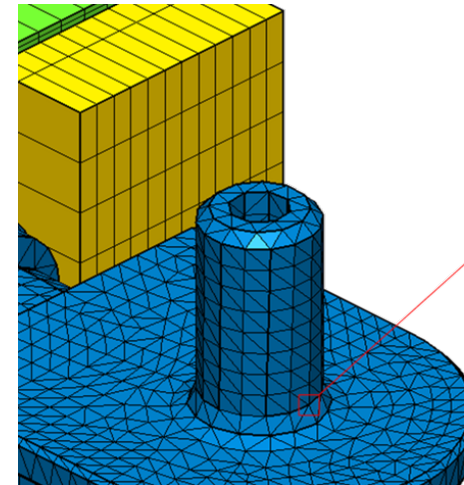


Dynamic loading by
sine function using
the self-weight in Z
direction



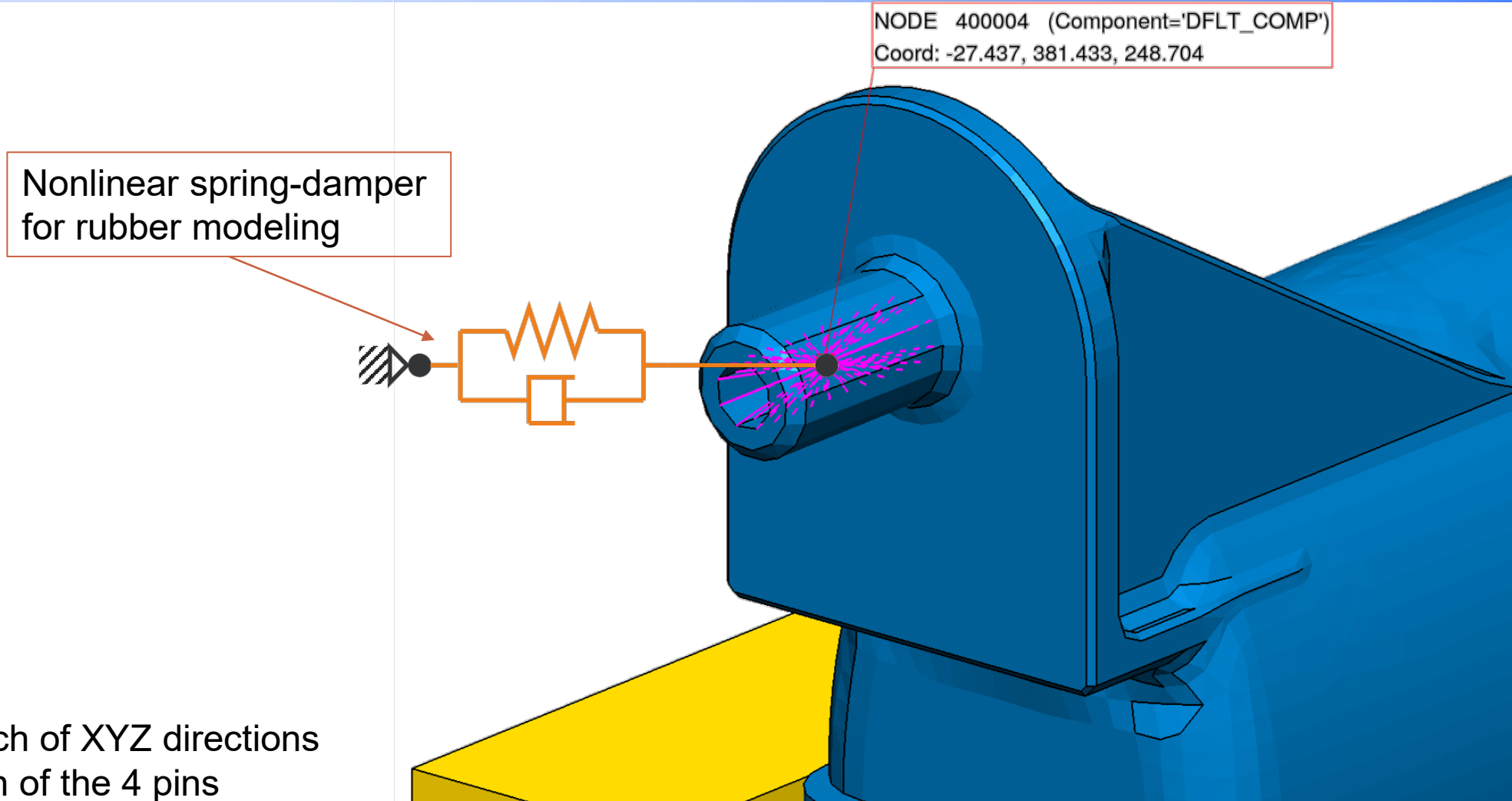
The node 328265 is
used to compare
the results.

NODE 328265 (Component='DFLT_COMP')
Coord: -27.906, 386.411, 241.704



NODE 328265 (Component='DFLT_COMP')
Coord: -27.906, 386.411, 241.704

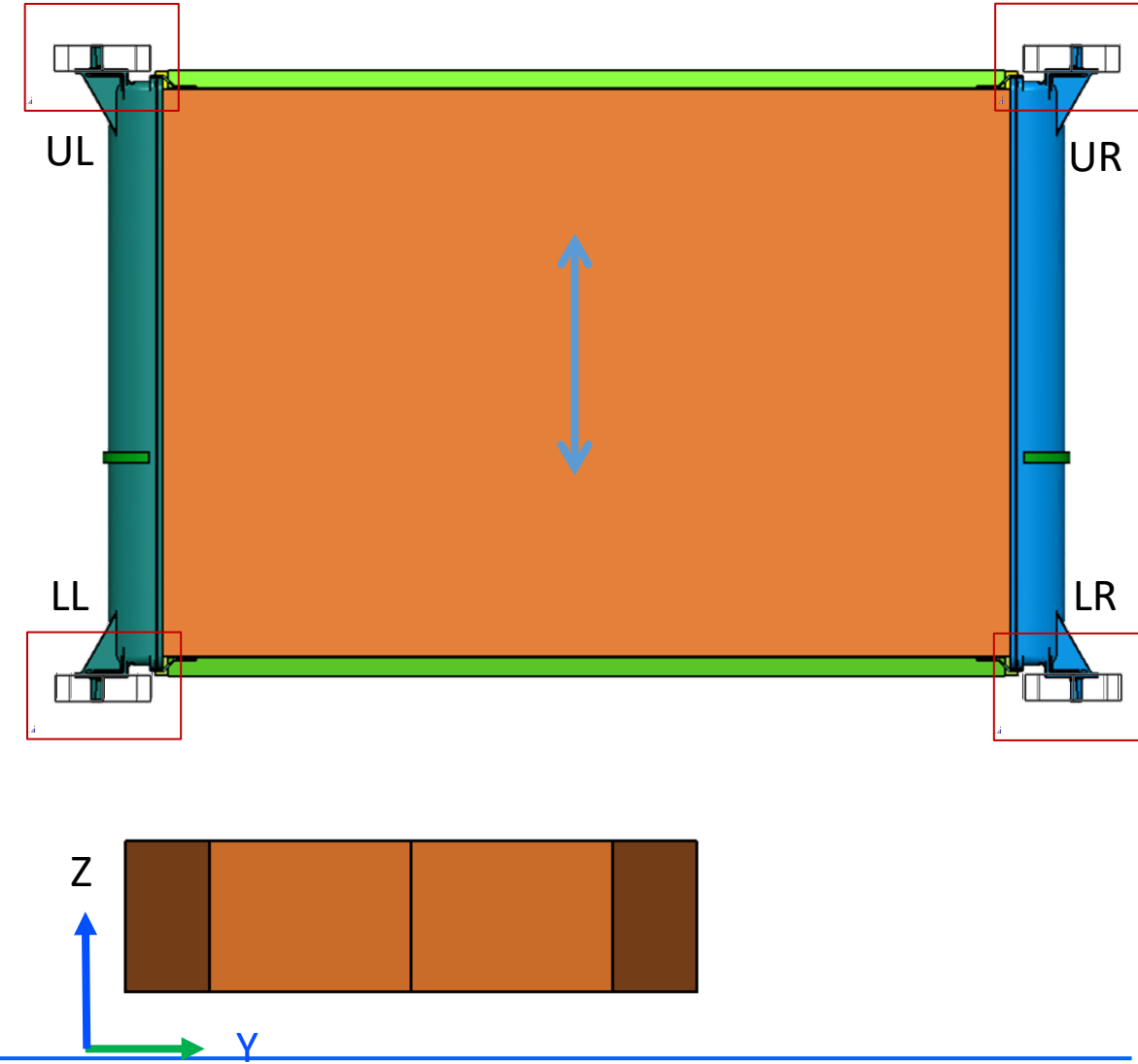
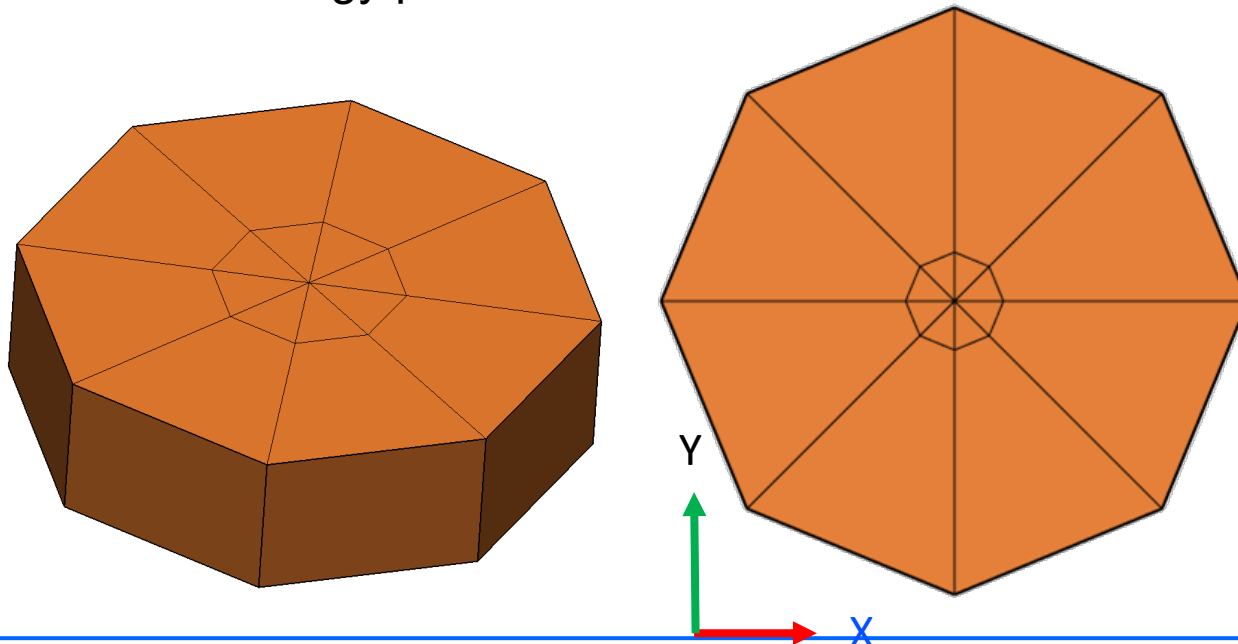
Modeling of Mount



- For each of XYZ directions
- At each of the 4 pins
- Both nodes have the same position

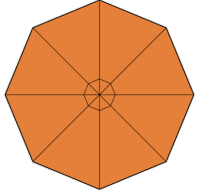
Excursus: Material Curves by Simulation

- Excursus: Material curves by simulation.
- Solid model with HEXE8 and PENTA6
- Diameter 60 mm, height 16 mm (like pin)
- DOF at border are suppressed
- 18 mm prescribed displacement of center node
- Reaction force for prescribed displacements
- Selected rubber material model: reduced polynomial strain energy potential

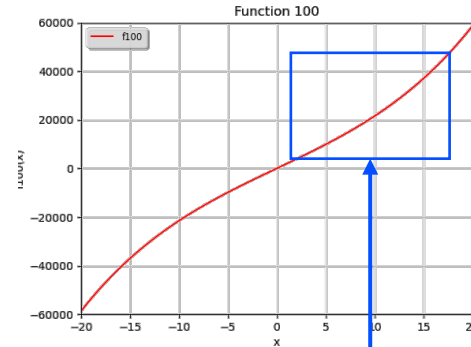


Fitting of Polynomial to Match Reaction Forces

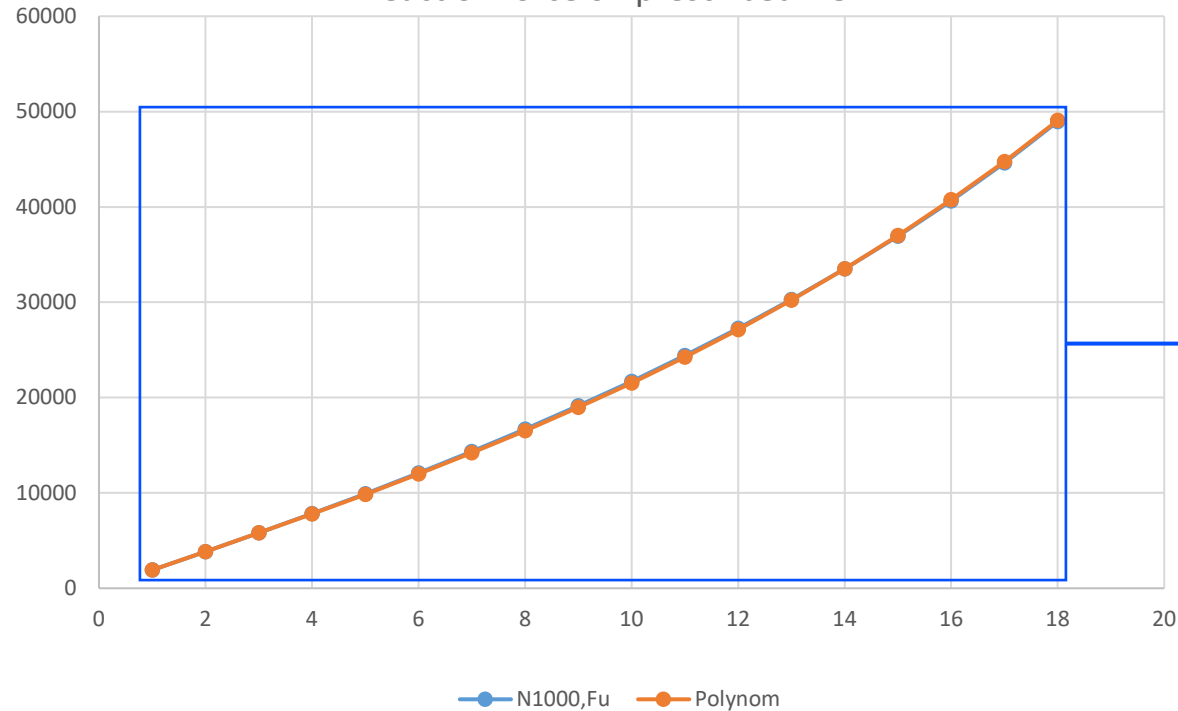
X, Y



$$f(x) = \underbrace{1917 \cdot x}_{\text{linear part}} + \underbrace{2.36 \cdot x^3 + 0.00047 \cdot x^5}_{\text{nonlinear part}}$$



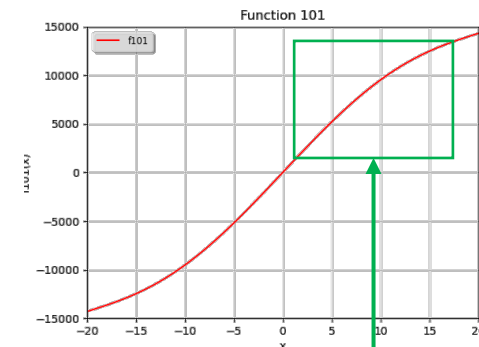
Reaction force on prescribed DOF



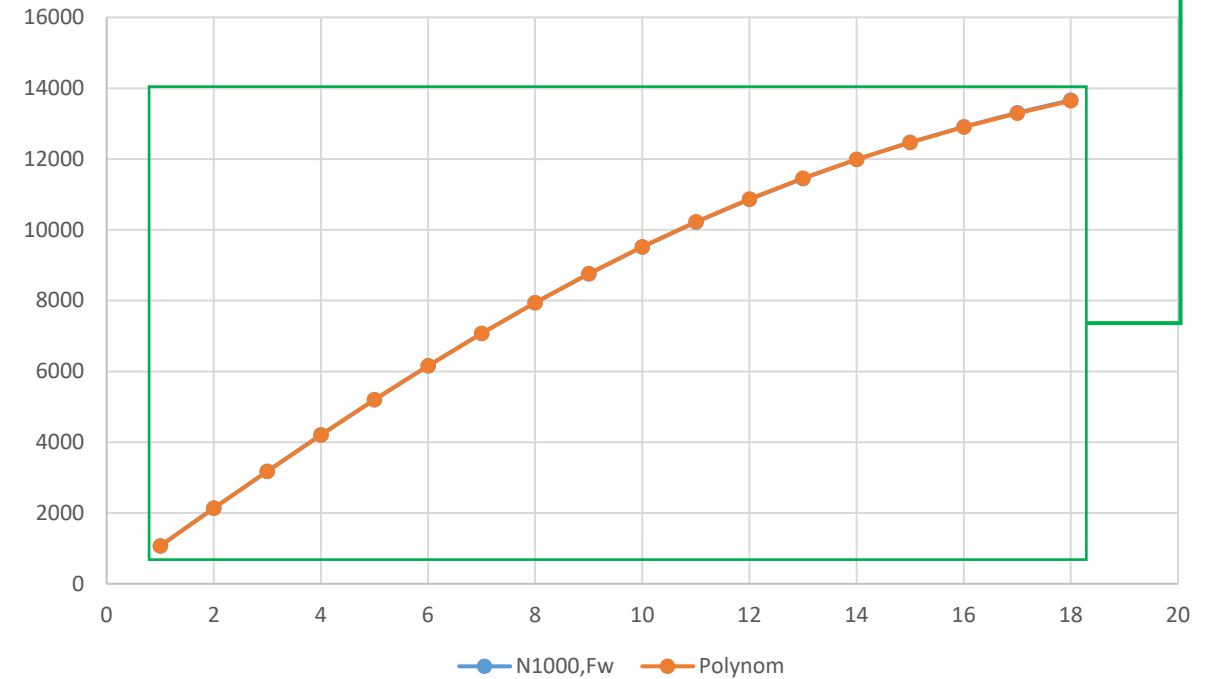
Z



$$f(x) = \underbrace{1071 \cdot x}_{\text{linear part}} - \underbrace{1.29 \cdot x^3 + 0.001 \cdot x^5}_{\text{nonlinear part}}$$



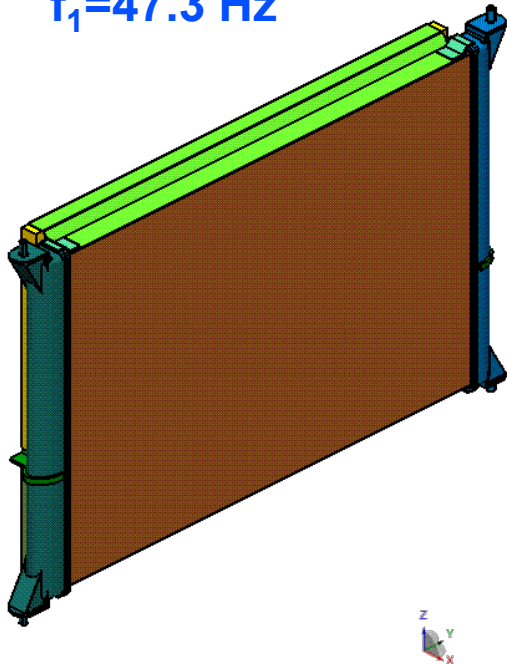
Reaction force on prescribed DOF



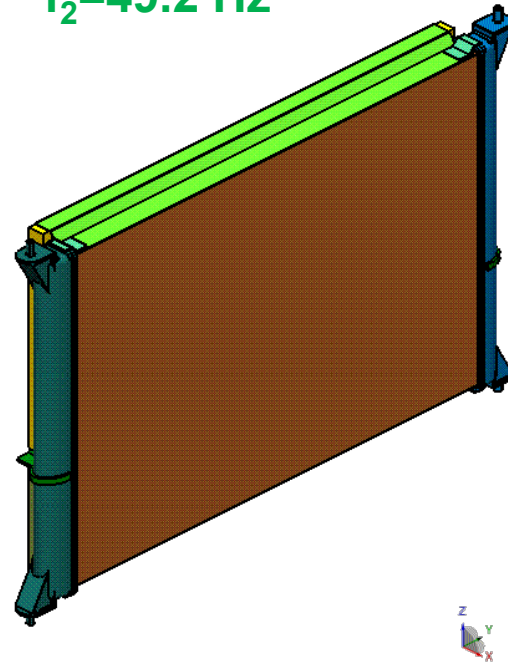
Eigenfrequencies of the Linear System

Mode	Frequency	EffM_U [%]	EffM_V [%]	EffM_W [%]	EffM_PHIU %	EffM_PHIV %	EffM_PHIW %
1	4.72755E+01	0.05	0.00	99.72	0.08	12.09	0.00
2	4.91918E+01	99.80	0.00	0.05	0.00	0.07	0.12
3	5.58942E+01	0.00	99.86	0.00	0.03	0.00	6.12
4	9.02673E+01	0.00	0.00	0.23	0.00	87.37	0.00
5	9.29685E+01	0.00	0.00	0.00	99.87	0.00	0.01
6	9.72080E+01	0.00	0.13	0.00	0.01	0.00	93.61

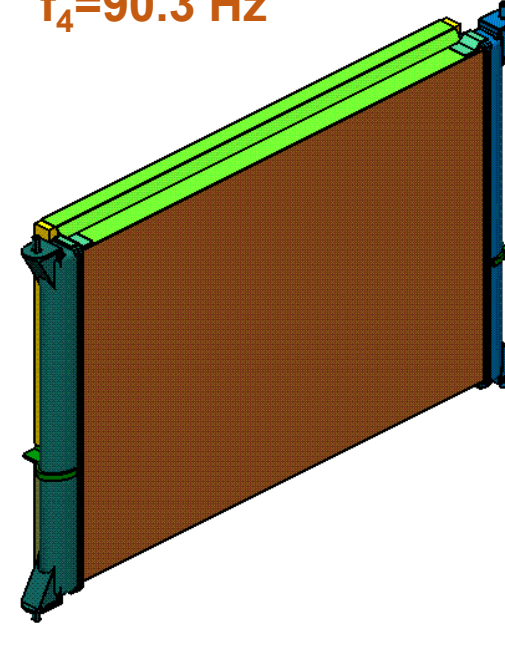
$f_1=47.3$ Hz



$f_2=49.2$ Hz



$f_4=90.3$ Hz



Linear spring force-deflection functions:

- XY : $f(x) = 1912 \cdot x$
- Z : $f(x) = 1071 \cdot x$

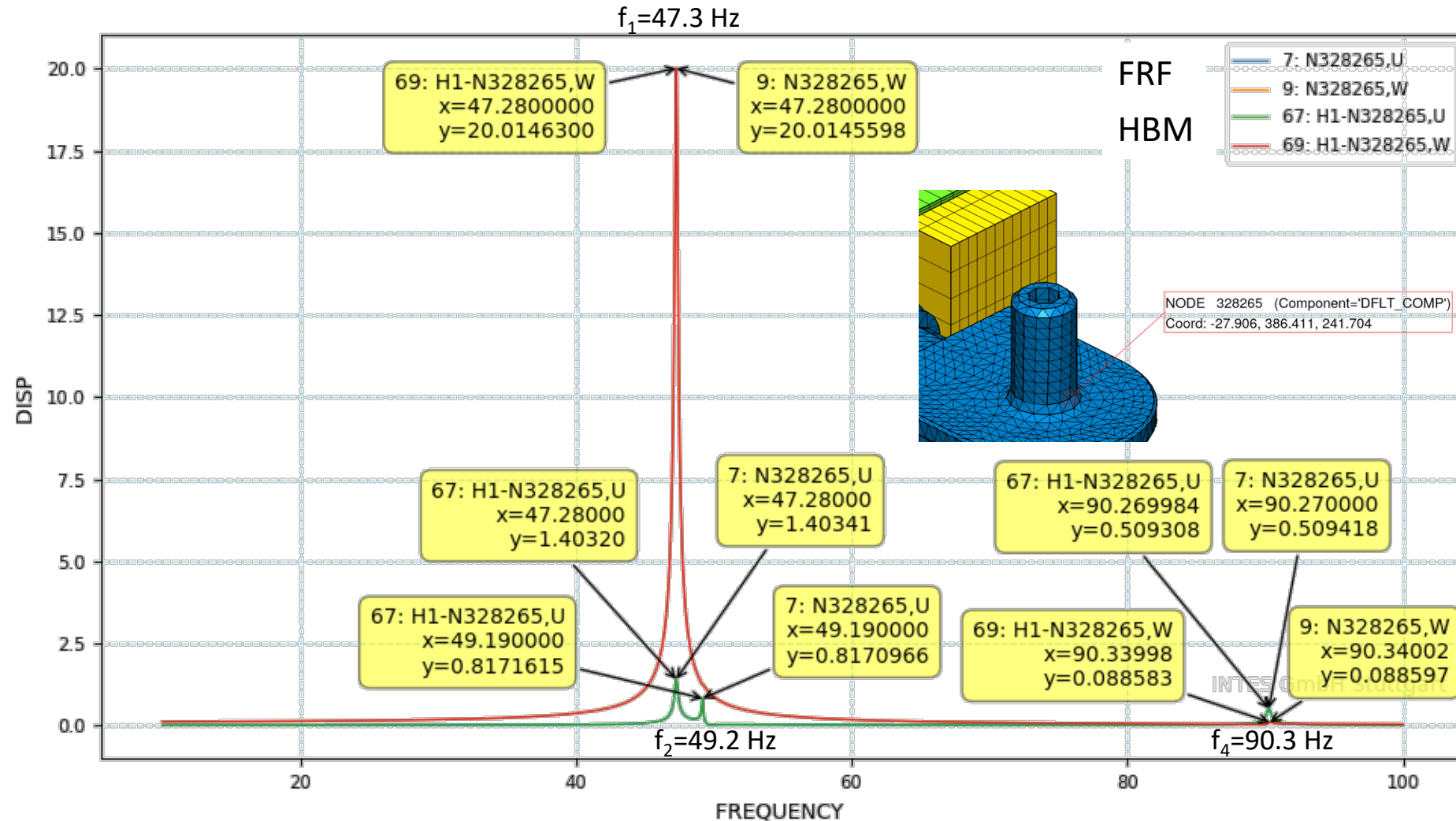
Center of gravity has not $x=0$:
→ response in X

Modes 1, 2 and 4 are excited!

Modes 3, 5, and 6 are not activated by the loading in Z :
→ no resonance in the FRF!

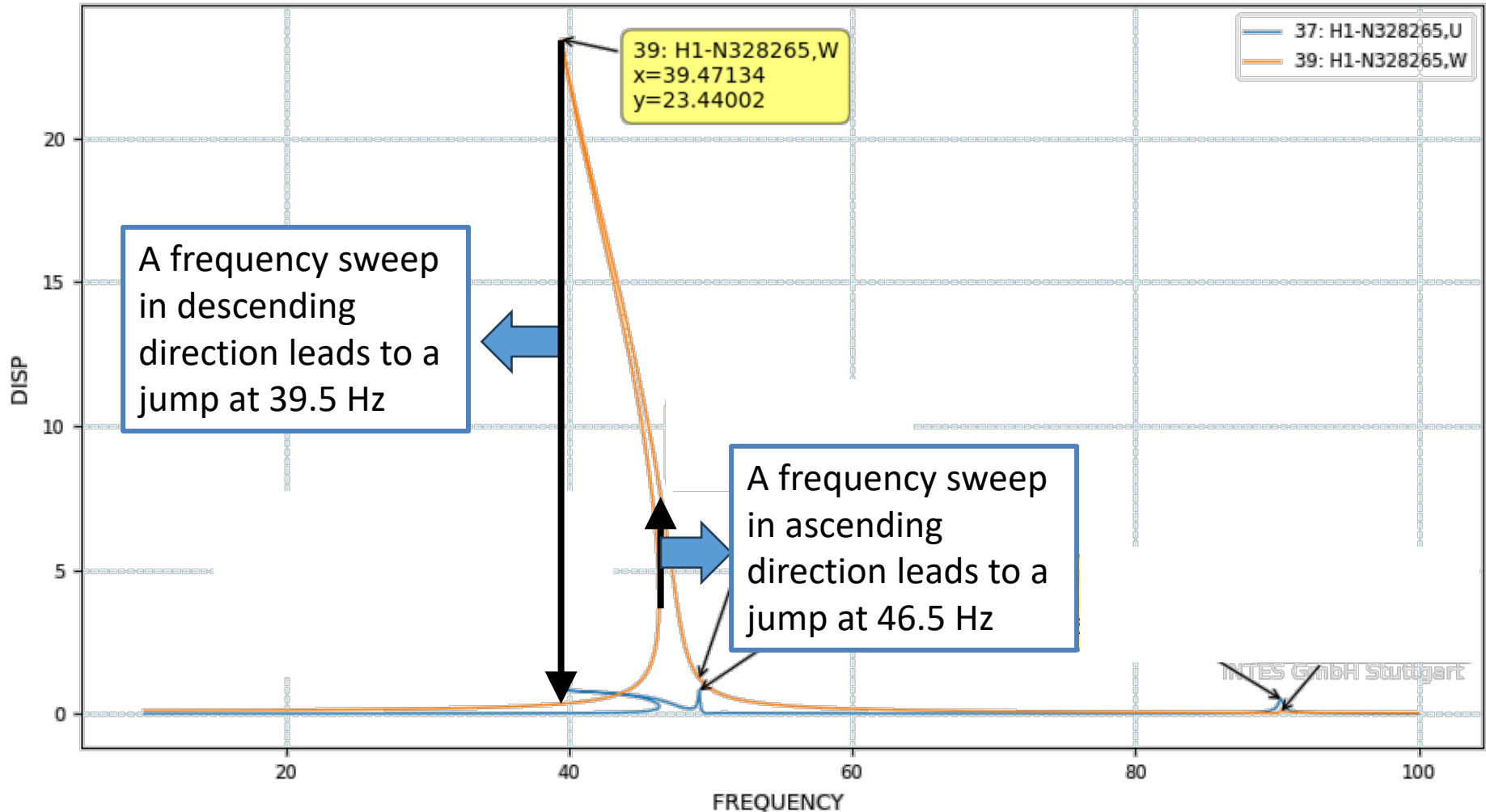
Linear Solutions with FRF and HBM

- **Linear** functions for rubber material:
→ results of FRF and HBM have to be the same.
- **Linear** part of functions:
XY: $f(x) = 1912 \cdot x$
Z: $f(x) = 1071 \cdot x$
- A linear damping function is used:
XYZ: $d(v) = 0.02 \cdot v$
- Comparison of linear FRF with the first harmonic of the HBM.
- **The results are the same!**



Nonlinear Solution with HBM

- **Nonlinear** rubber material:
 XY: $f(x) = 1917 \cdot x + 2.36 \cdot x^3 + 0.00047 \cdot x^5$
 Z: $f(x) = 1071 \cdot x - 1.29 \cdot x^3 + 0.001 \cdot x^5$
- Multiple solutions for same frequency
 → jumps in the response and may cause instabilities.
- The damper coefficient remains at 0.02.



Sweep with Jump

https://www.youtube.com/watch?v=zclp8vLKJzU&ab_channel=NG_Design

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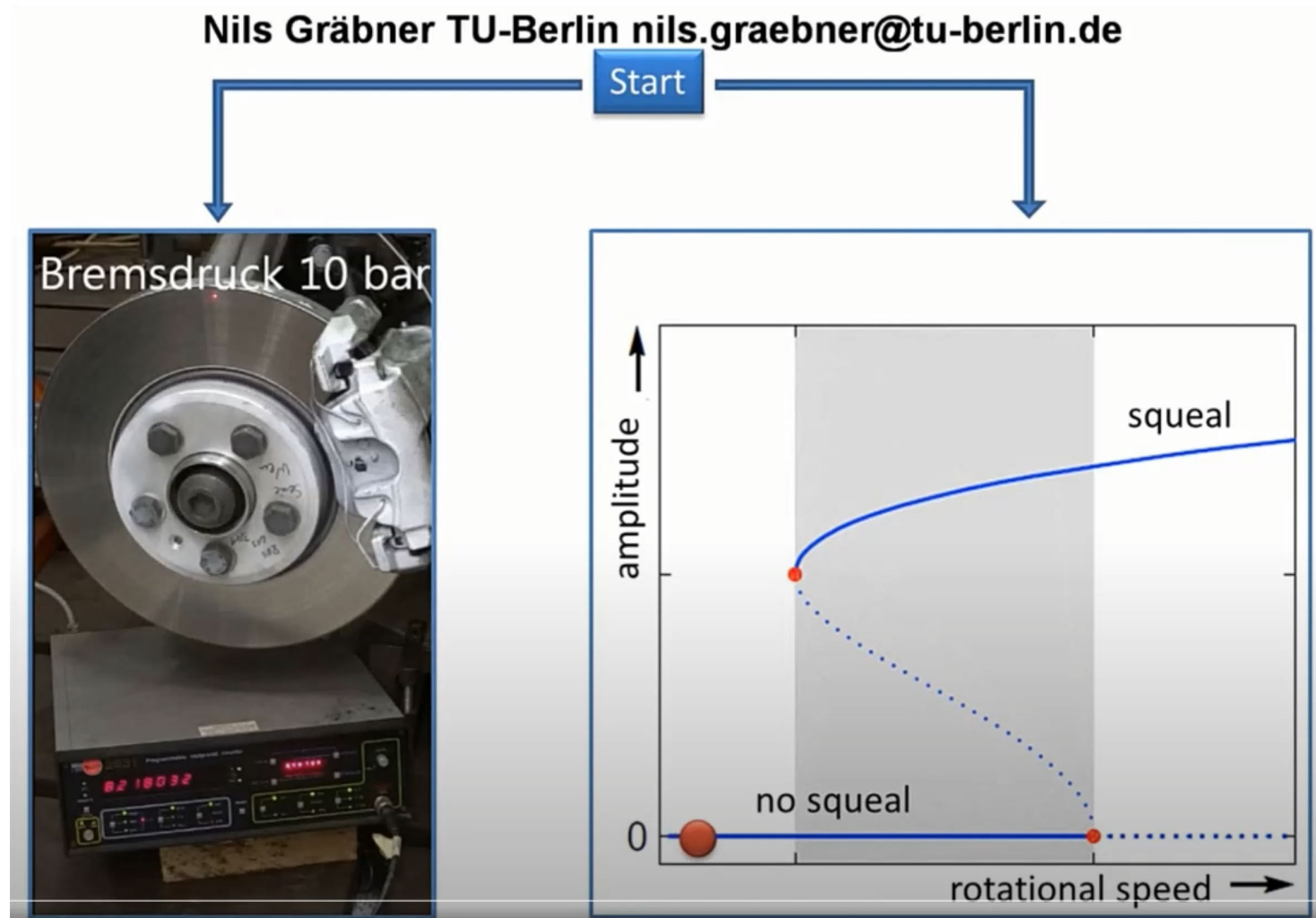
EB2013-NVH-022

Advances in simulation of brake squeal

Supported by:
 Federal Ministry
of Economics
and Technology
on the basis of a decision
by the German Bundestag

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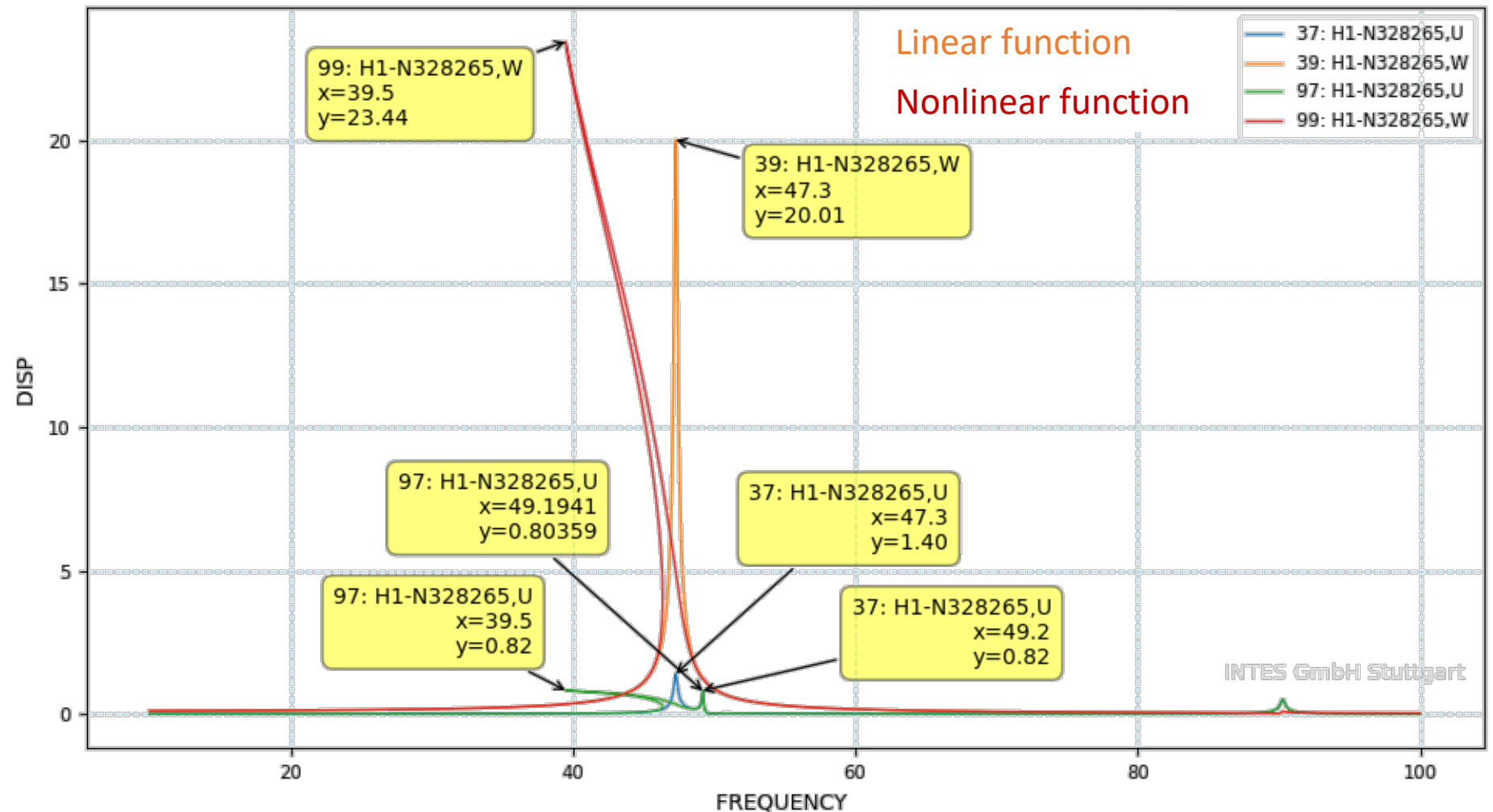
Comparison of Linear and Nonlinear Solution

The **nonlinear** solution shows:

- higher amplitude
- lower frequency at peaks

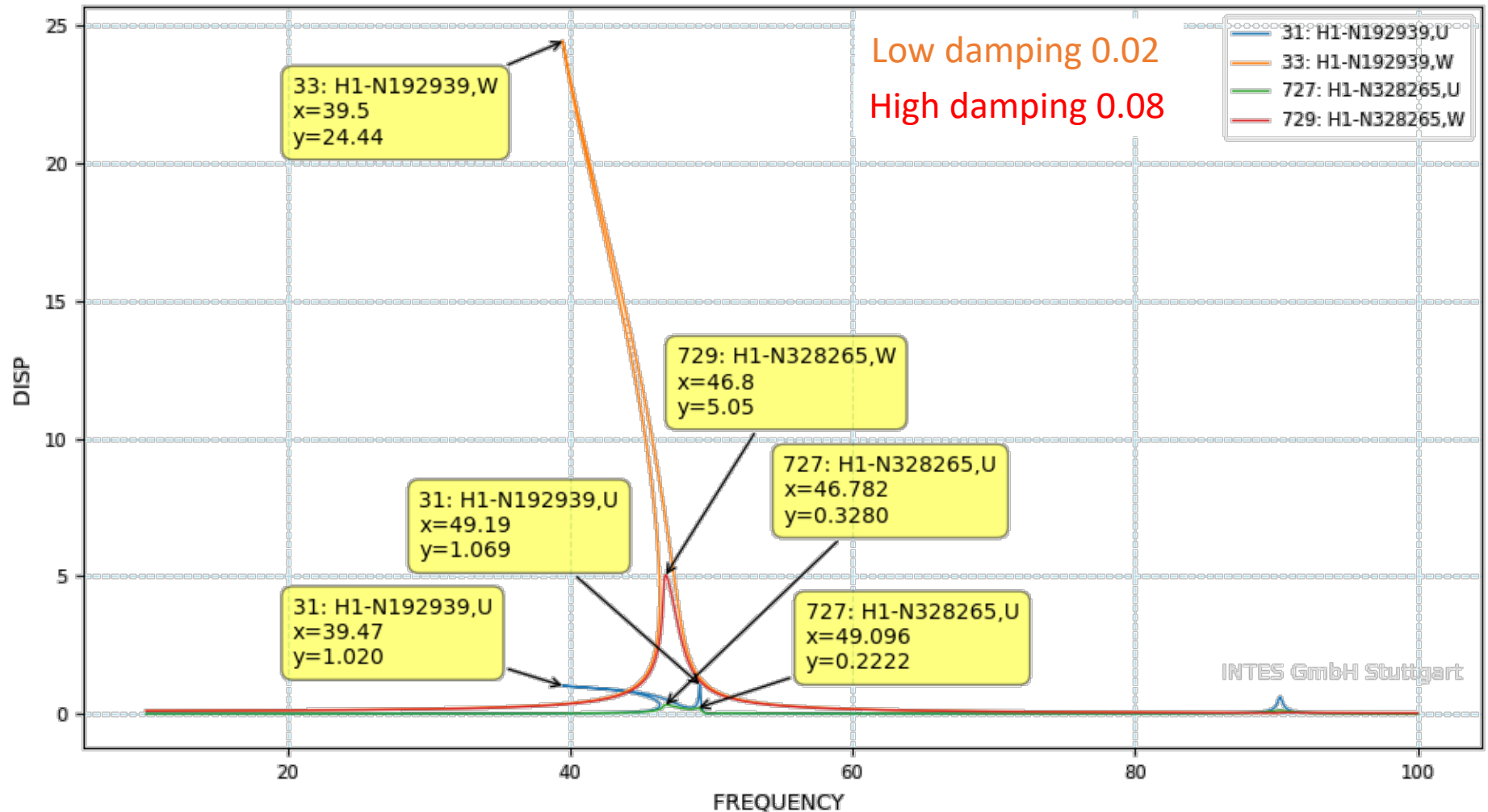
Multiple solutions for one frequency could indicate instabilities:

- Technical products should avoid instabilities (multiple solutions).
- This can be achieved by increasing the damping of the rubber mount.
(The used rubber material should be able to provide this higher damping.)



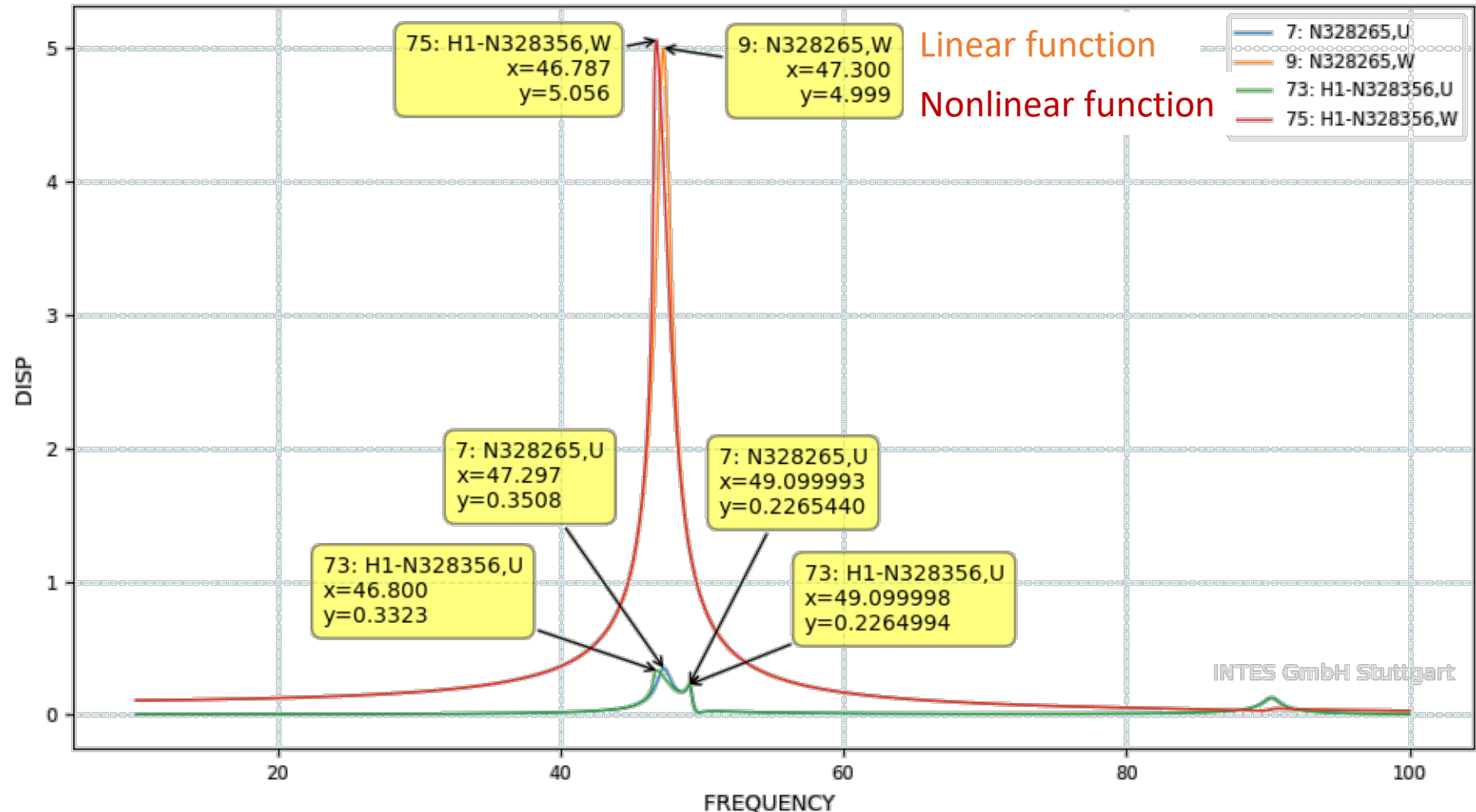
Effect of Higher Damping

- The **damping** is changed from **2%** to **8%**
- Rubber is capable to provide 30% damping.
- The nonlinear spring force-deflection functions remain unchanged.
- No multiple solutions at one frequency with higher damping. No possible instability.

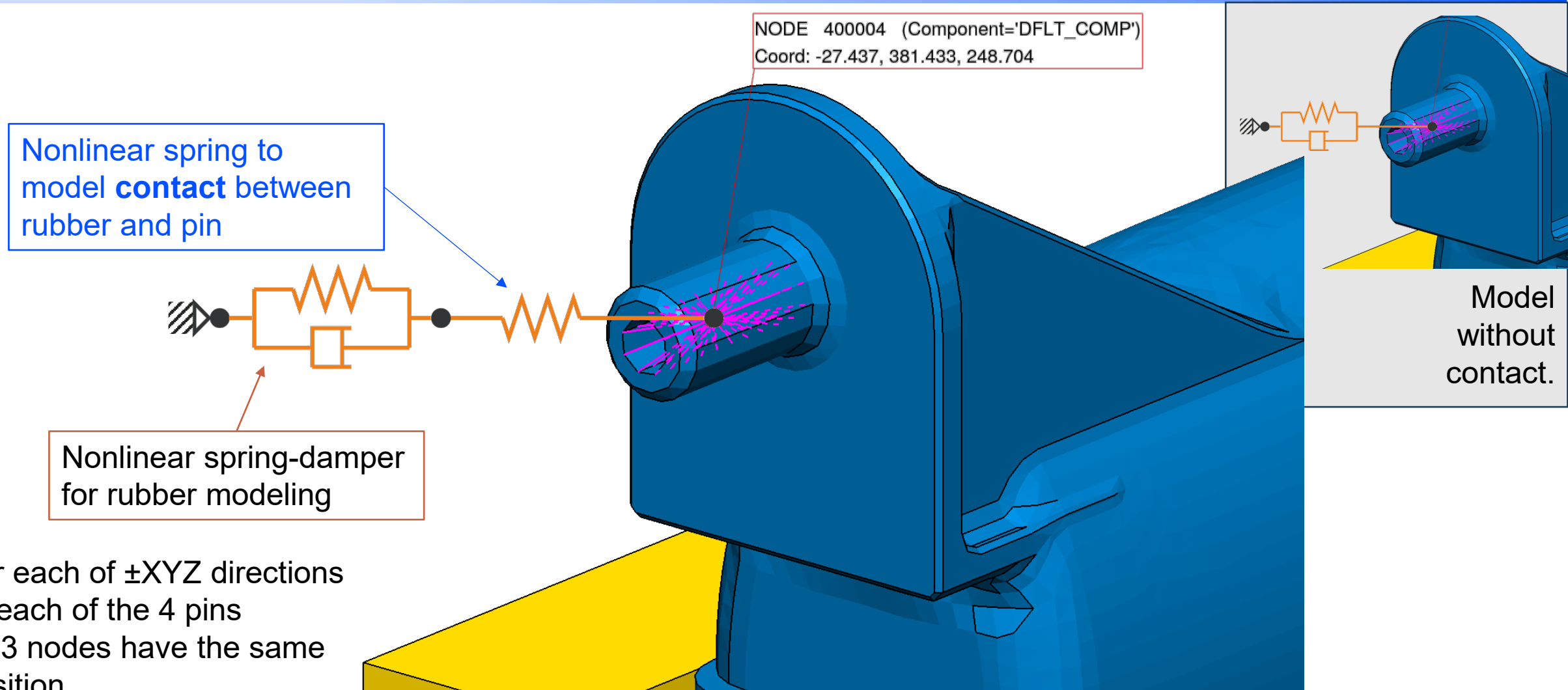


Comparison of Linear and Nonlinear Solution with Higher Damping

- The nonlinear function result in a reduced first resonance frequency compared to the linear function (47.3 Hz → 46.8 Hz)
- Same damper coefficient (0.08).
- The amplitudes are about the same.

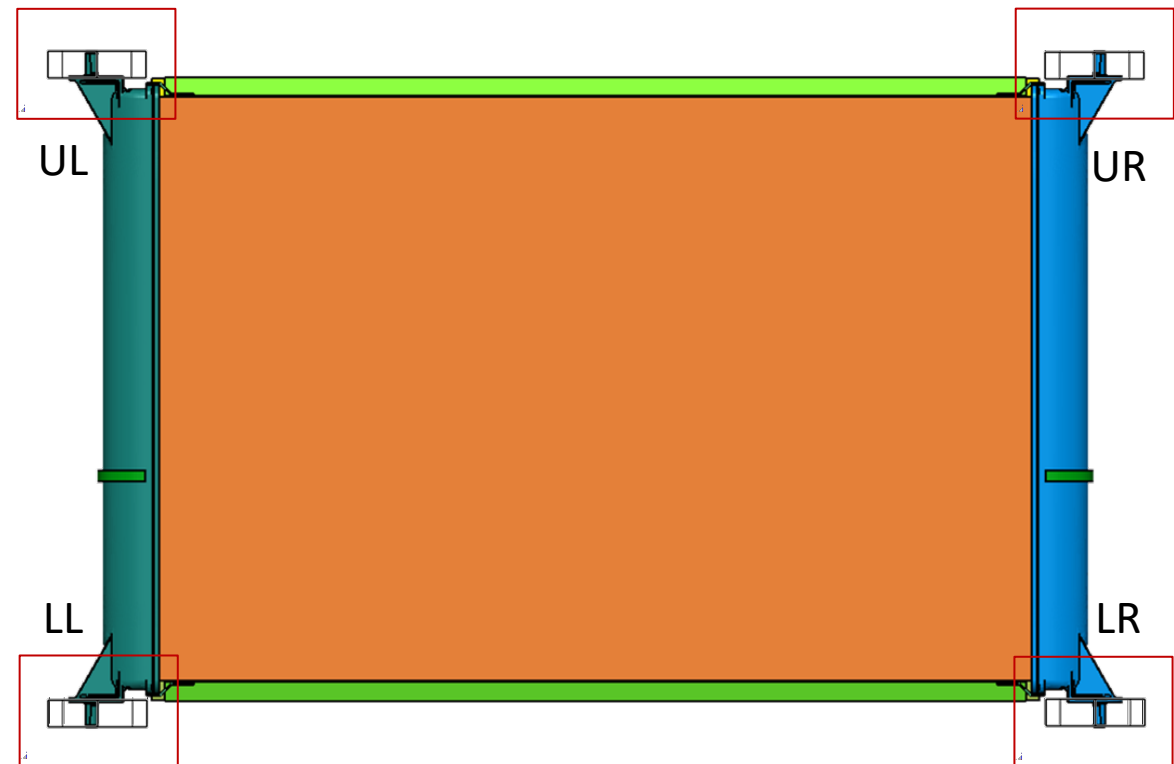
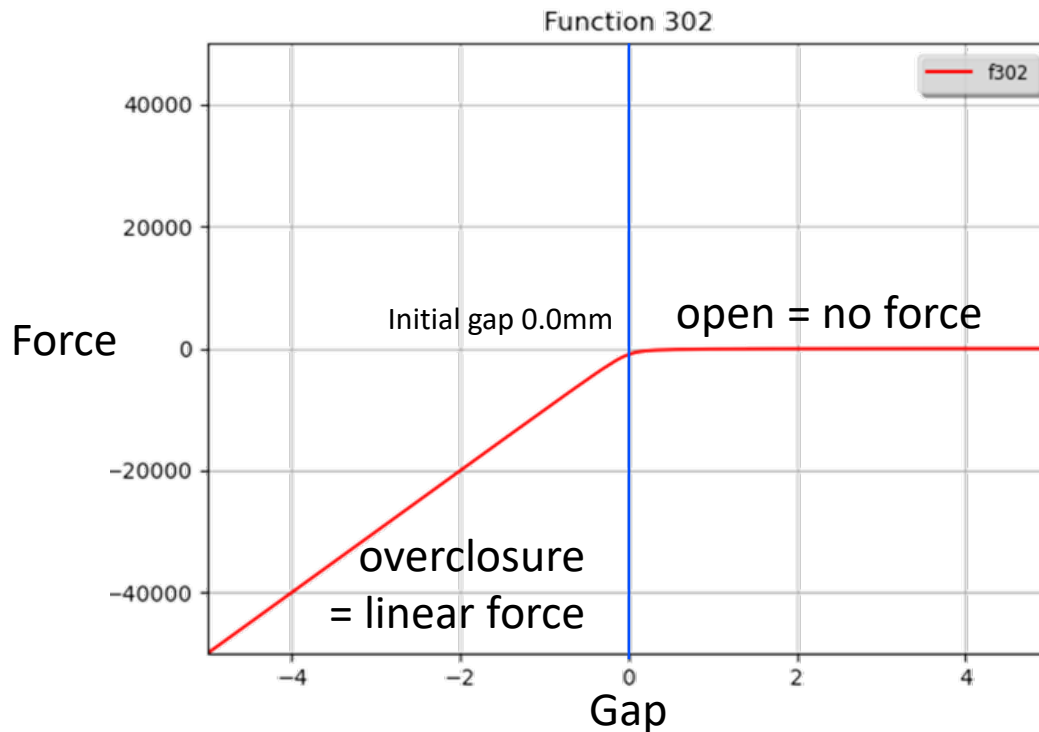


Modeling of Mount with Contact



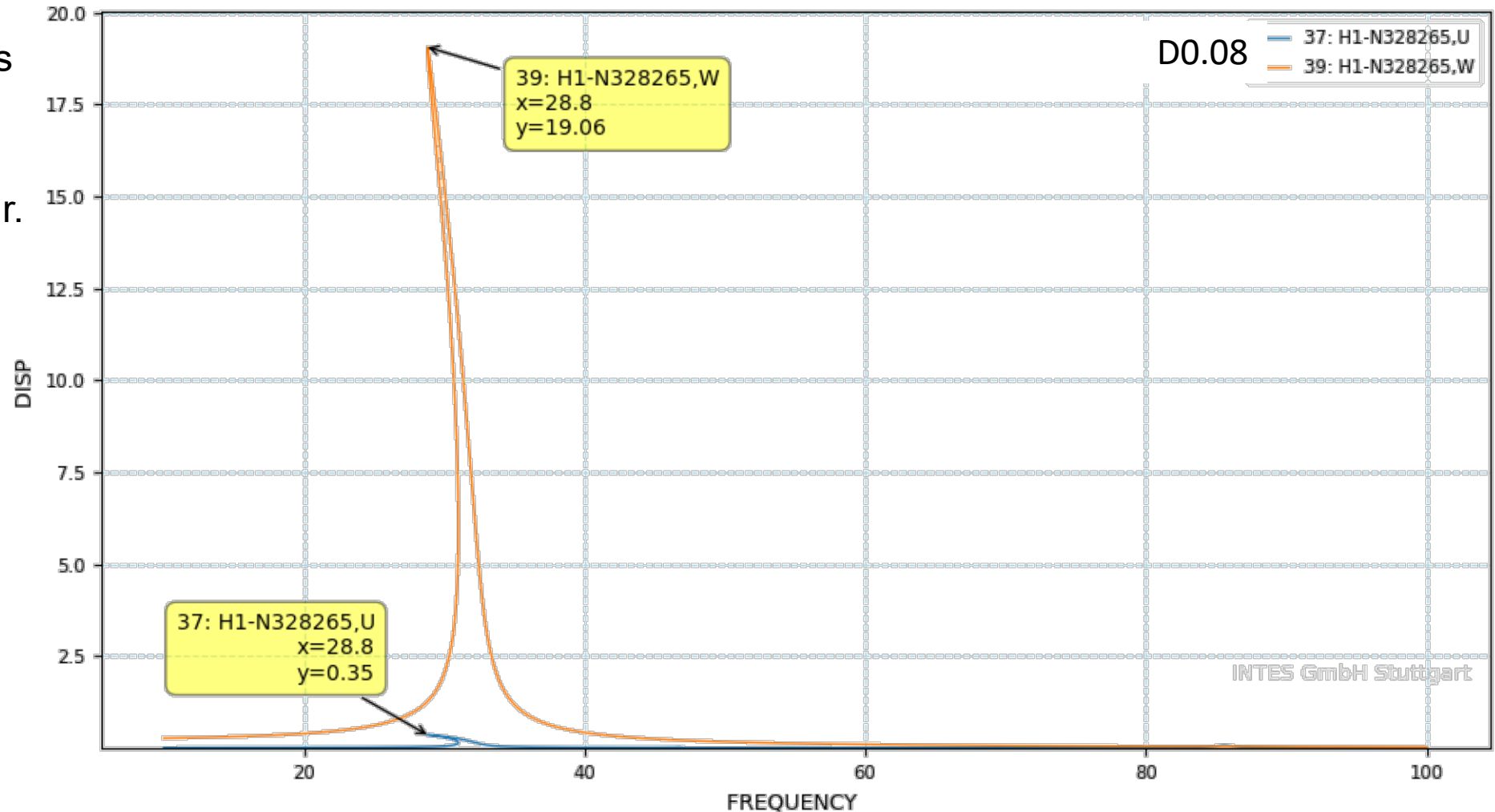
Zero Initial Gap Width for Lowest Nonlinearities

- Assumption: **no initial gap** (the radiator is not loose but properly fixed in the rubber mounts)
→ **all initial gap widths are set to Zero.**
- To avoid multiple solutions the damping coefficient of the rubber mounts has to be increased from 0.08 to **0.2. This is still in the range of possible rubber damping values (≤ 0.3).**



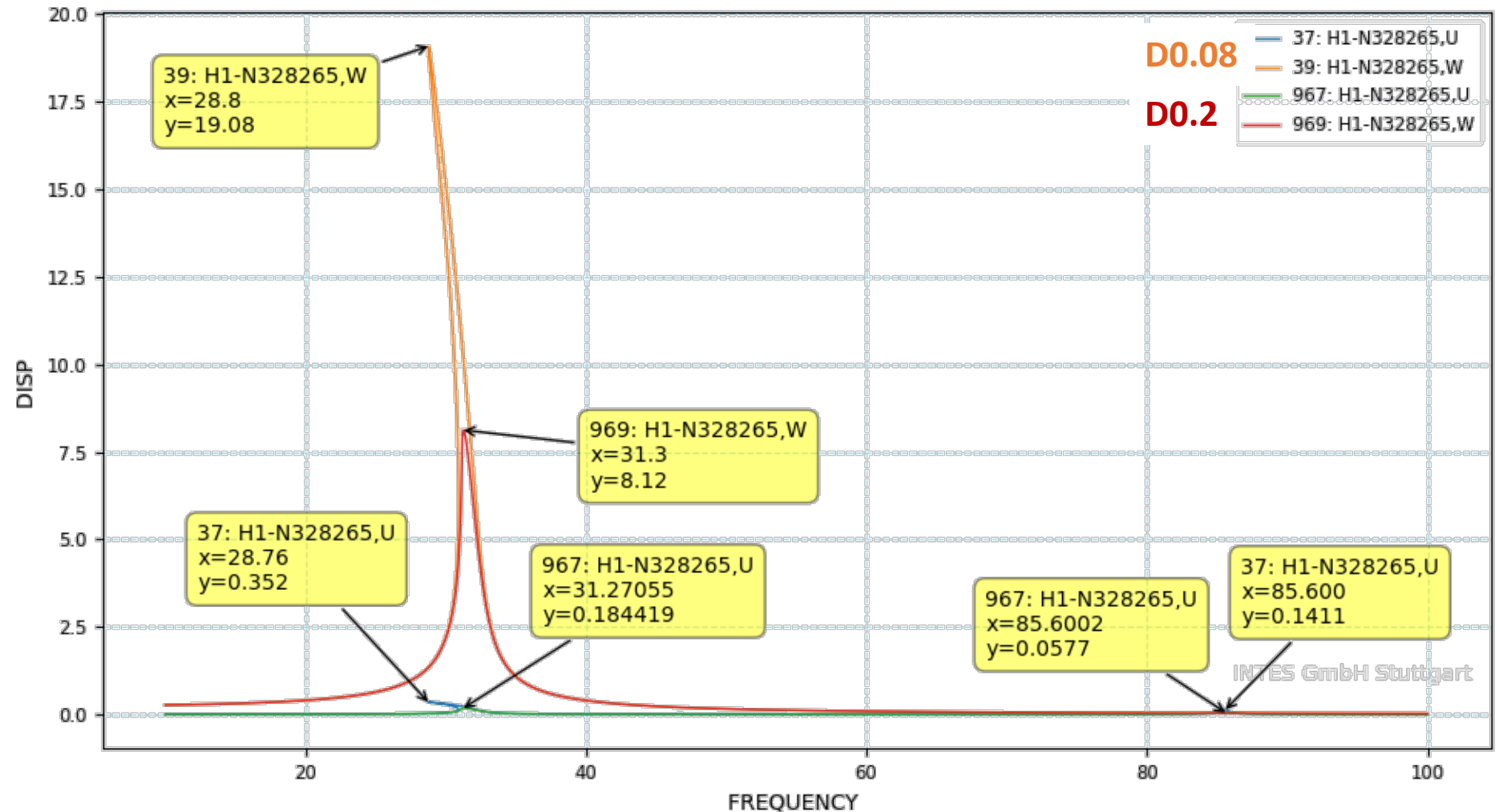
Response with Contact and Rubber Damping 0.08

- Damping coefficient is 0.08.
- Multiple solutions for same frequency occur.



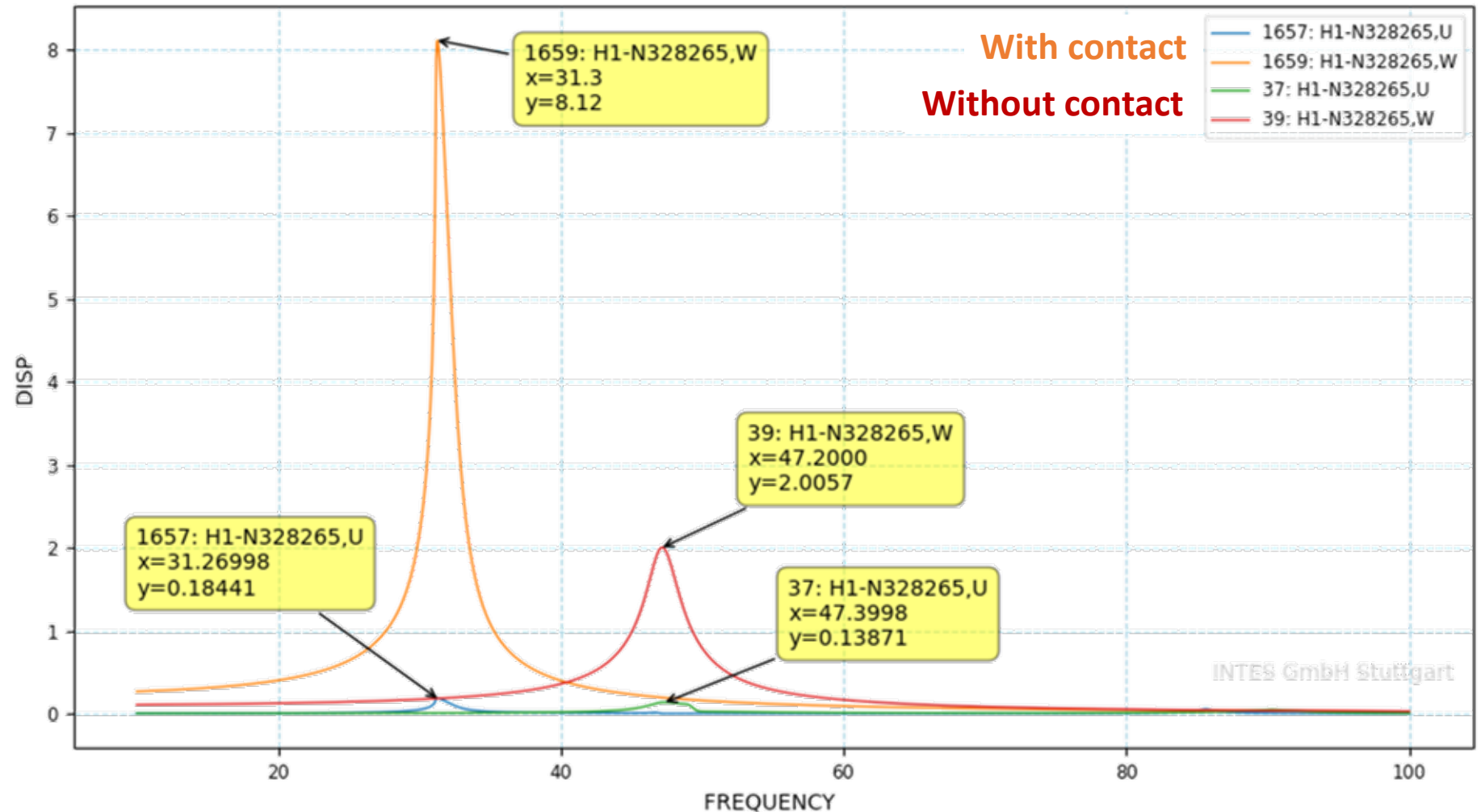
Comparison of Response with Rubber Damping 0.08 and 0.2

- Damping coefficient is 0.08 and 0.2.
- No multiple results anymore at any frequency for damping 0.2.



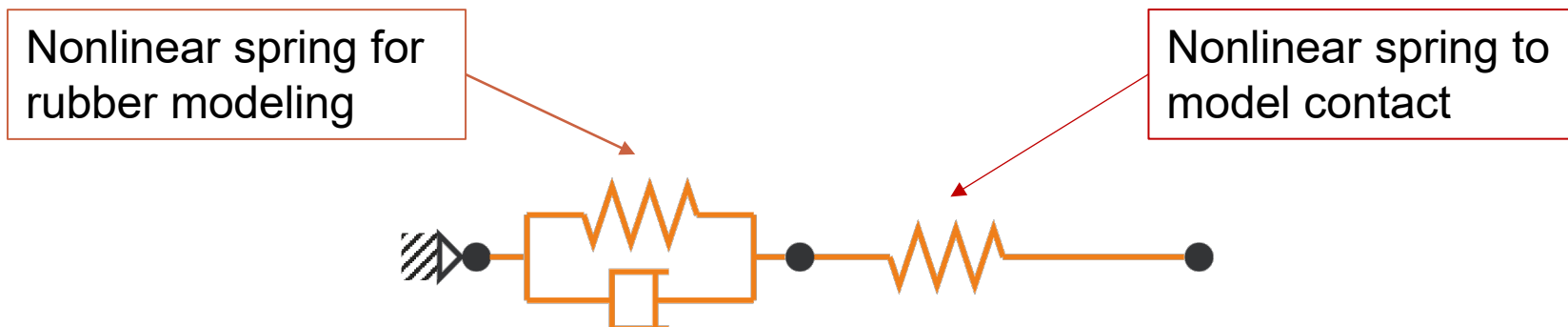
Comparison of Response with and without Contact

- Damping coefficient is 0.2 for both.
- The main resonance peak is now at **31.3 Hz** with contact and at **47.2 Hz** without contact.
- The maximum amplitude is **8 mm** with contact and **2 mm** without contact.
- If the mounting cannot avoid contact, the contact case is more critical.



Transformation to Time Domain

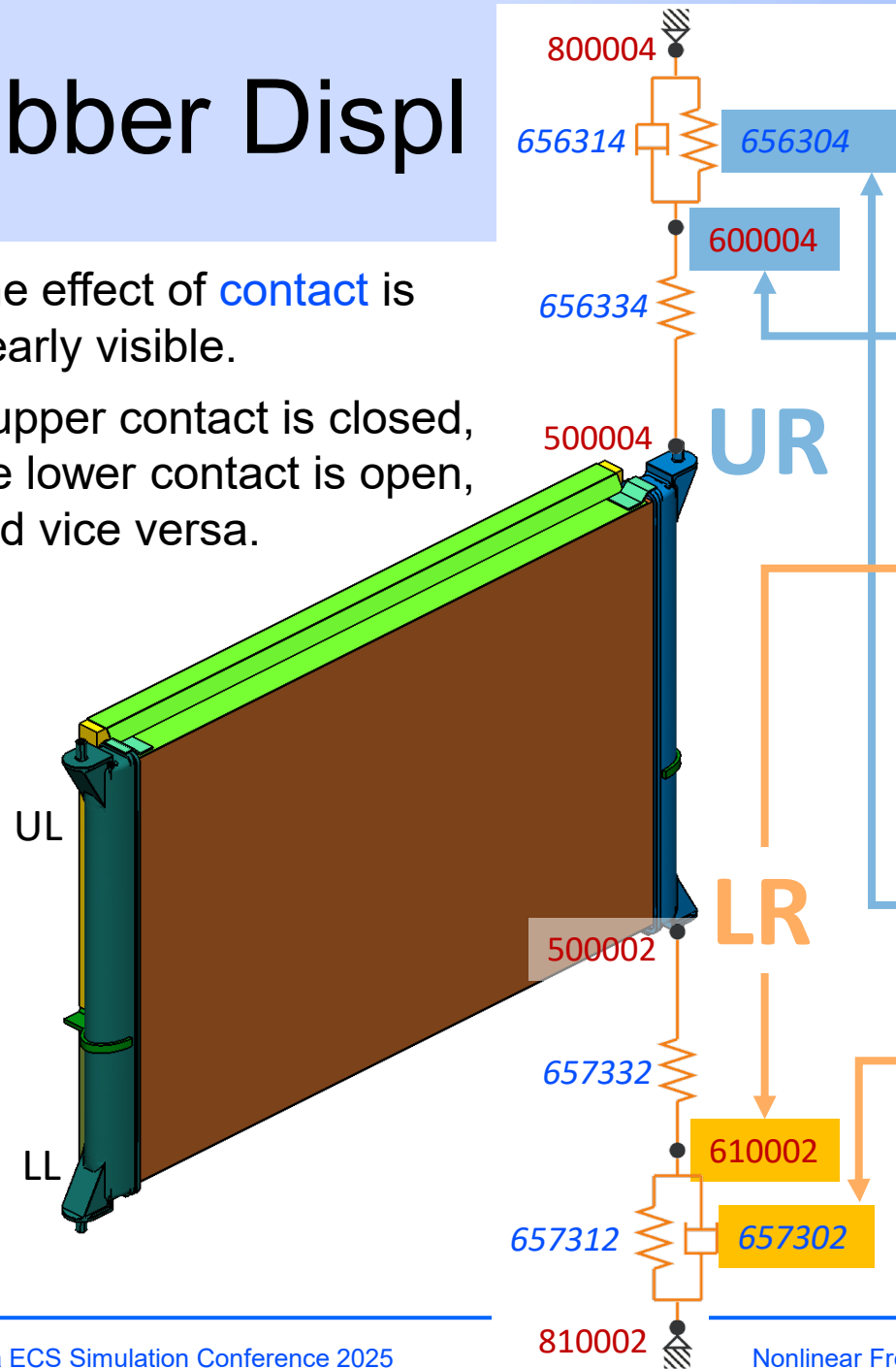
- The results of HBM are in frequency domain.
- Transformation to time domain is available.
- To perform the transformation a frequency (from HBM analysis), a time step and the number of loops have to be selected. Here:
 - Frequency of the highest resonance peak at 31.3 Hz.
 - The time step is $1./(100 \cdot 31.3) = 0.0003195$ (100 loops at 31,3 Hz).
 - With 200 time steps and the step size 0.0003195, one gets two sine waves at 31.3 Hz.
- In time domain we can check the spring elements for rubber and contact.



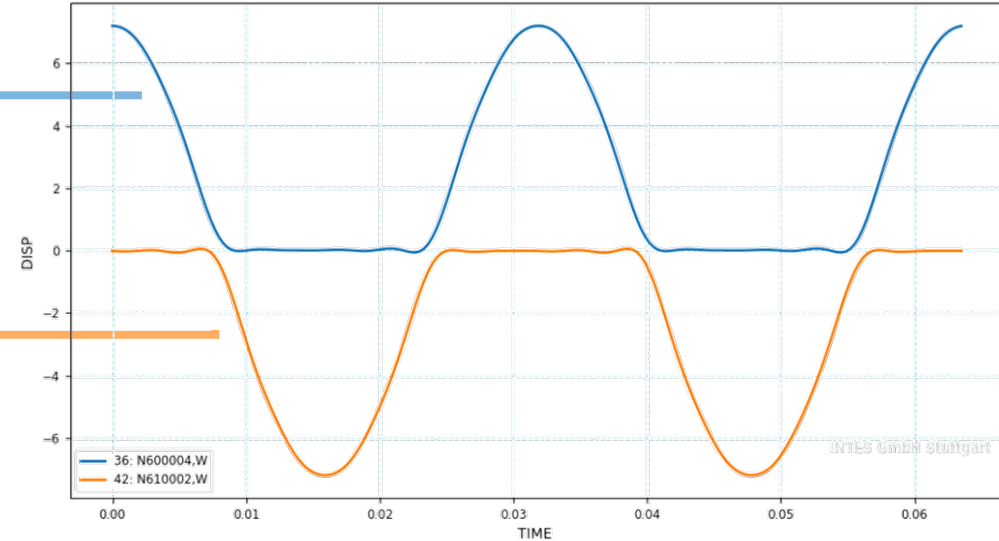
Vibration (color: vMises stress)

Rubber Displ

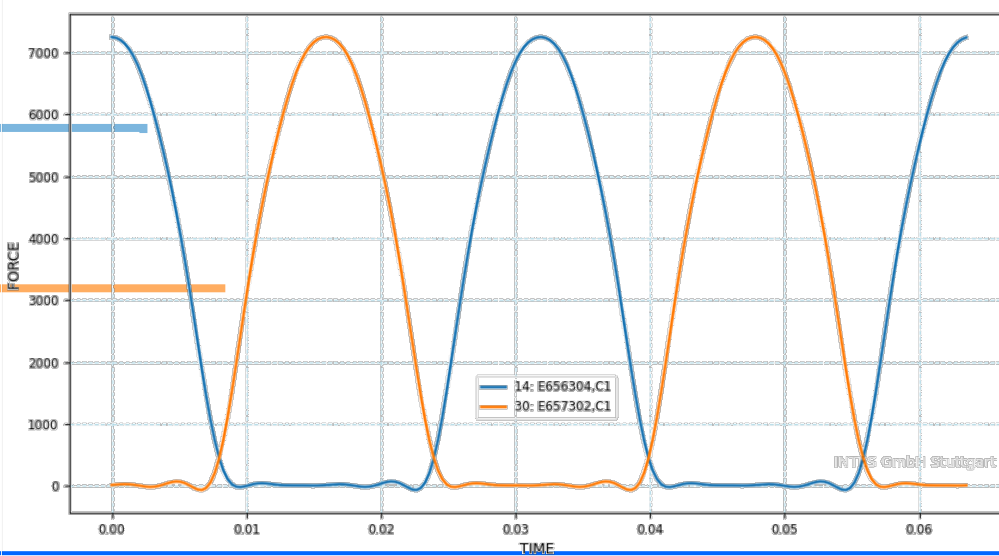
- The effect of **contact** is clearly visible.
- If upper contact is closed, the lower contact is open, and vice versa.



acement and Force in Z

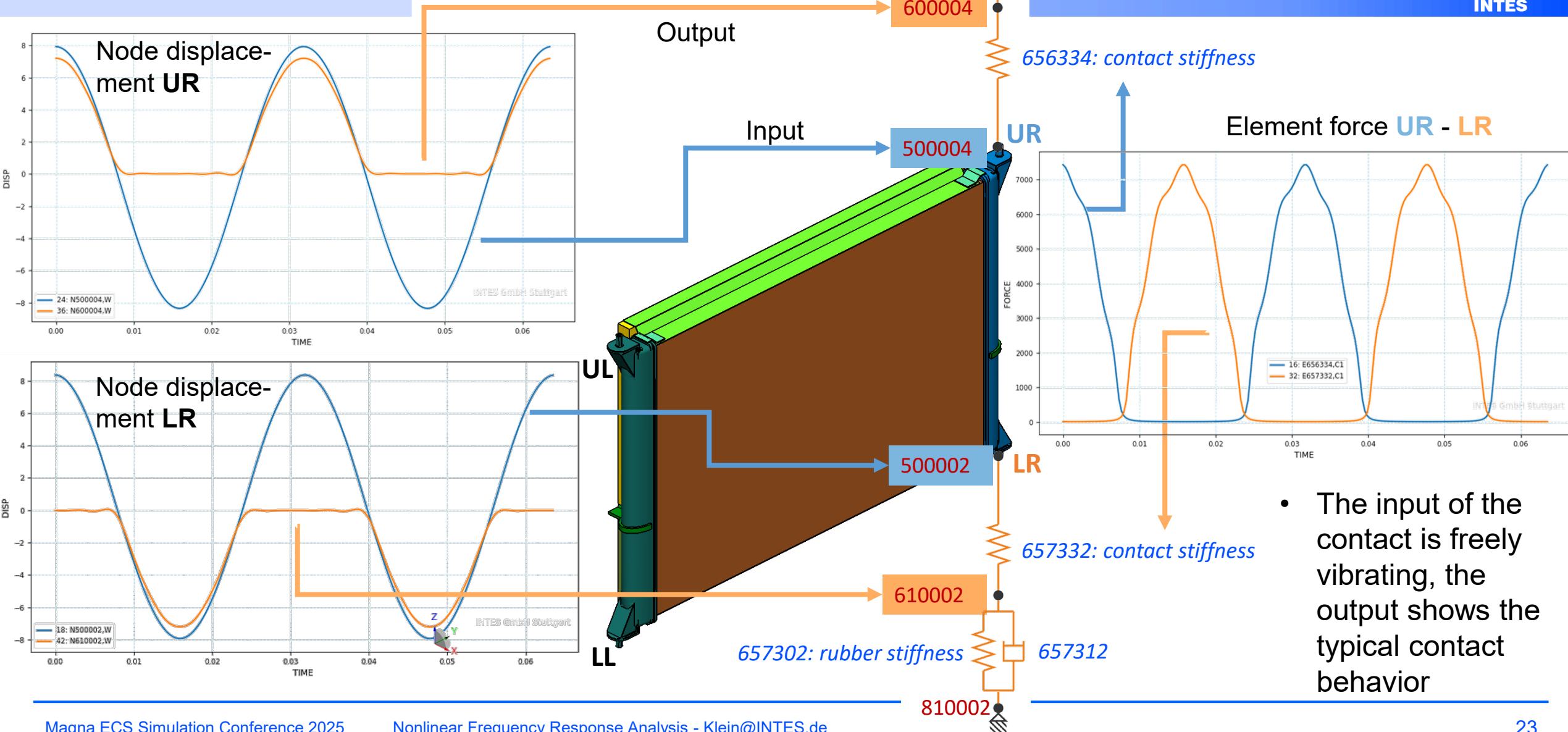


Node displacement
UR - LR



Element force
UR - LR

Contact Displacement and Force in Z



1. HBM is used to solve nonlinear dynamic tasks in frequency domain.
2. Nonlinearities used for the mounting of a radiator-like structure are rubber mounts and contacts at the 4 fixation pins.
3. HBM results detects multiple solutions for same frequency, to avoid instabilities sufficient damping was used.
4. After transformation the results are available in time domain.
5. Result check in time domain shows contact behavior.
6. HBM, transformation in time domain, nonlinear time-history are all integrated in PERMAS.
7. Analysis sequence possible in a single job.
8. Excellent run time for HBM in comparison to steady state for each frequency in time domain.

Thank You!

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